## Introduction

Six decades after astronomer Hawkins (1963, 1964) suggested that Stonehenge was a Neolithic computer, little consensus has been reached regarding Stonehenge's role in astronomy. By reconstructing and reinterpreting $40.5^{\circ}$ geometrically, we demonstrate that Stonehenge was once an observatory equipped with rulers, protractors, and compasses shaped/scaled in lozenges.

$\begin{array}{ll}\text { Fig. } 1 \text { (Left). } & \text { Stonehenge: Aubrey } \\ \text { Holes } & \text { (Green), } \\ \text { Station Rectangle }\end{array}$ Holes (Green), Station Rectangle
(Blue), and the $40.5^{\circ}$ alignment (Red) of the Avenue with solstice sunrise and sunset
Fig. 2 (Below). Four Lozenges: (a) Bush Barrow Large Lozenge (BBL) $186 \times 155 \mathrm{~mm}$; (b) Bush Barrow Small lozenge (BBS): $31 \times 18.6 \mathrm{~mm}$ $155 \times 110 \mathrm{~mm}$; (d) Upton Lovell Gold Button (ULB): 47 mm in diameter, 7 mm in height


## Materials and Methods

Materials (publicly available data)

- Photos and scales of artifacts and sites (e.g., Figs. 1-2)
- Phases and dating info: 3100-1700BC

Astronomy and geometrical background
Methods

- Perform measurement and numerical experiments objectively
Search for significant angles, simple ratios, scale-free patterns, and logically necessary instruments such as a protractor
Design and test geometric models and keep simplifying them
Mix and match artifacts with sites across locations, periods, and scales


## Results

I. Matching for Cardinal Orientation

Bush Barrow Lozenge (BBL), "crown jewel of the King of Stonehenge," was unearthed within 800 m of Stonehenge. By finding a good pivot, we obtain $12^{\circ}$ multiples on BBL (Fig. 3). Meanwhile, the Sarsen Circle (Phase 3) locks BBL to the cardinal orientation and turns itself into a compass. Alternatively, by matching the Station Rectangle (Phase 3) with BBL's zig-zag vertices, the east-west direction is locked. Therefore, there is no need to assume prehistoric Britons encoded a Pythagorean Triple, 5:12:13, in the Station Rectangle.

1:1000


Fig. 4. Scaling: the 1:1000 scaling can be verified by the inscription of Aubrey Holes and by the outer bank of diameter 110m.

# The Double Meaning of $40.5^{\circ}$ on Stonehenge and Bush Barrow Lozenge 



Axis, Aubrey Holes, and Station Rectangle are all reflected in the lozenge structure

## From Matching to the Base Unit

By circumscribing Aubrey Holes with Clandon Barrow Lozenge (CBL), every interior ring of Stonehenge is circumscribed by a band on CBL (Fig. 4), suggesting they followed one common pattern. This matching is highly unlikely a coincidence due to the presence of its striking scale factor- 1:1000. Consequently, a base unit is hinted at, which is further reinforced by the common length between the long axis of CBL and the short axis of BBL (Fig. 5). BBL and CBL thus become rulers for this base unit. Because of this common length, 0.155 m , we call this unit one Lozenge.


Fig. 5 . Base Unit of Length: the matching length, 0.155 m (Blue) qualifies BB and CBL as rulers; and the $40.5^{\circ}$ alignment (Red) on both BBL and CBL

In the unit of Lozenges, BBS is scaled at 0.2 UBL is scaled at 0.3 , the gap between Sarsen stones is 20 , and the gap between Aubrey holes is $10 \pi$. In addition, the Megalithic Yard, MY, is ( $2 \pi$ ) tan $40.5^{\circ}$ Lozenges. Factor $\pi$ indicates the use of ropes looping around circular tree trunks, drums, or wheels with diameters in Lozenges, which could have been a practical feature for large sites.

III. Nested Square Pattern

We propose a nested square pattern (Fig. 6) to characterize both layers and angles of the lozenges in Fig. 2. Some derived forms are shown in Fig. 7.
Algebraically: $X_{1}=1 ; X_{2}=\frac{\left(X_{1}+X_{3}\right)}{2} ; X_{3}=\frac{1}{\sqrt{2}} ; X_{i}=\frac{X_{i-2}}{\sqrt{2}}$ for $i \geq 4$
Numerically: $X=\{1 ; 0.854 ; 0.707 ; 0.604 ; 0.5 ; 0.427$; 0.3536; 0.302; 0.25;...

Approximately: $X_{1}: X_{2}: X_{3}=7: 6: 5 ; X_{4}: X_{1} \approx 3 / 5$; $X_{7}: X_{2} \approx 5 / 12$

## Trigonometrically: $\quad \theta=\arctan (X)$

$\theta_{1}=45^{\circ} \quad$ models a square
$\theta_{2}=40.48^{\circ}$ models Bush Barrow Large, BBL $\theta_{3}=35.26^{\circ}$ models Clandon Barrow Lozenge, CBL $\theta_{4}=31.13^{\circ}$ models Bush Barrow Small, BBS


## Conclusion

$40.5^{\circ}$ means the solstice sunrise/sunset alignment at Latitude $51.2^{\circ}$; it also means a latitude-independent geometric pattern including a family of special angles and ratios between layers. Implied in this pattern were a base unit of length, 0.155 m ; the Base- 2 and Base-10 scaling; measuring devices: ruler, protractor, and compass; and all the logical fundamentals: star gazing, time keeping, geometrical reasoning, design thinking, learning, and innovating. Profoundly intertwined, the double meaning of $40.5^{\circ}$ uncovers Stonehenge's pivotal role as an astronomical observatory \& standard bearer in the prehistoric world through its siting, design, construction, and operation.

| Lockyer | $(1906)$ |
| :--- | ---: |
| Hawkins | $(1963,1964)$ |
| Thom | $(1967,1974)$ |
| Hoyle | $(1977)$ |
| Taylor | $(1980)$ |
| Lawson | $(9997)$ |
| Doutre | $(2001,2003)$ |
| North | $(2007)$ |
| Johnson | $(2008)$ |
| Maumené | $(2017)$ |
| lliffe | $(2019)$ |
| Jiang and Jiang | $(2021)$ |

IV. Protractor Mechanism

Inspired by a 1300 BC Egyptian protractor, we reverse engineered a protractor mechanism that involves eccentric pivoting (Fig. 8). This mechanism measures all 360 angles at a $1^{\circ}$ accuracy. Thus, BBL and CBL were both rulers and protractors. By aligning the instruments with the cardinal directionś, this indicates a compass.

 independently, where the subtraction is done by pivoting. The near-integer values in the $\theta$-sequence enable the $1^{\circ}$ precision

## V. Beyond Stonehenge

Further insights are gained by testing th "lozenge model" including the angle ( $40.5^{\circ}$ ) \& length ( 0.155 m ) against additional Megalithic artifacts and sites. One critical test is Woodhenge, a contemporary of Sarsen Circle (Phase 3) within a two-mile distance.


Fig. 9. Woodhenge: we model the site plan as two sets of concentric circles. cross points along BBL's $40.5^{\circ}$ diagonal. By further circumscribing both sites

By circumscribing Woodhenge with lozenges, we discovered a 1:2 scale ratio between these two sites This ratio not only verifies Woodhenge's conformance in orientation and scale, but also generates a resonance with Stonehenge to enlighten a lasting tradition passed down from the Neolithic Era to the Bronze Age. Q.E.D.

